CONCLUSIONS

The analysis presented extends an available steady diffusion solution [5] to the solution of a time-dependent heat conduction problem. In so doing it provides a theoretical basis for the invariability of the rate of loss of heat by conduction in wedges, namely, the difference between the rate of loss of heat per unit time, per unit depth along the two bounding planes, from the wedge surface and that from a corresponding surface of a semi-infinite solid being independent of the time variable. The analysis has found important applications in the area of casting solidification [7, 8].

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HEAT TRANSFER TO HORIZONTAL TUBES IN A PILOT-SCALE FLUIDIZED BED

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NOMENCLATURE

C_R,	correction factor used by Wender and Cooper [8];
D_T,	outer diameter of tube;
$\bar{d}_p$,	surface-to-volume mean particle diameter;
H,	expanded bed height;
H_{mf},	bed height at minimum fluidization;
h_b,	bed-to-exterior time mean heat transfer coefficient;
h_{max},	maximum value of h_b as U increases;
U,	superficial gas velocity;
U_{mf},	superficial gas velocity at minimum fluidization;
ε,	overall bed voidage;
ε_{mf},	bed voidage at minimum fluidization.

INTRODUCTION

While many data have been published on heat transfer between immersed surfaces and gas fluidized beds, there are relatively few results for large-scale systems and for superficial gas velocities typical of industrial beds. During our investigation of heat transfer to horizontal tubes in the freeboard region above fluidized beds [1], some results were also obtained for immersed tubes. These data, reported in this communication, are for a reasonably large column and at gas velocities and temperatures and entrained solids recycle conditions more representative of industrial practice than most previously reported data.

METHODS

The experimental column, constructed of stainless steel, was 0.254×0.432 m in cross-section and 3.0 m high. Quartz windows allowed observation and filming of the bed and surface behaviour. The column was heated externally by inconel tubular heaters of total power 21 kW braised to the walls of the column. The operating bed temperature was always in the range 385–425 K. A bundle of horizontal tubes of 25.4 mm o.d. was present in the column in four rows of four tubes each. All tubes were internally finned and made of copper with external chrome-plating to prolong life and reduce absorptivity to radiation. The centre of the lowest row of tubes was 0.76 m above the gas distributor. The vertical centre-to-centre separation of successive rows was 3.0 D_T. The tubes spanned the 0.254 column dimension.

One of the 16 tubes was instrumented with eight chromel–constantan thermocouples embedded in the tube wall flush with its outer surface. A differential thermocouple was used to measure the temperature rise of silicone oil coolant circulated through the tube from a constant temperature bath system. Measurements of temperature difference down to 0.5 K could be made with a reproducibility of $\pm$ 2.4% or better. Disc-and-doughnut type mixers, one at either end, forced the oil to mix before its temperature was measured by the differential thermocouple. This instrumented tube could be placed in any of the 16 tube locations. (In practice, it was always located at one of the 8 interior positions, i.e. with tubes on either side, in order to minimize any influence of the containing walls. The remaining 15 tubes were water cooled. The heat transfer coefficient for the instrumented tube was obtained by numerical integration, with corrections for conduction losses to the bed wall through the insulating fiberfrax rope. Radiation never amounted to more than 1% and was therefore neglected. Both the overall (bed-to-oil) and outside (bed-to-exterior-surface) heat transfer coefficients were measured for each run. A separate experiment was carried out to determine the inside transfer resistance. Further testing of the accuracy of the system was obtained with particle-free air in cross-flow. Very good agreement was obtained [1] between the time–mean overall heat transfer coefficients for the instrumented tube in each of the rows and the well known correlations of McAdams [2] and Morgan [3].

The gas distributor was of the multi-orifice variety. Pressure taps were provided at intervals of 0.12 to 0.14 m above the distributor. The static bed height was 0.75 m for most of the results shown here. Mean expanded bed heights were determined from axial pressure profiles. An external cyclone connected to a hopper and dipleg provided for recycle of entrained solids to the region just above the distributor. Experiments were carried out with three batches of silica sand whose properties are given in Table 1.

Pressure fluctuation traces obtained with a pressure transducer and visual observations of the bed showed that the bed never reached the turbulent region of fluidization for the range of gas velocities (up to 1.3 m/s) used in this work. While the top surface of the bed fluctuated considerably, these fluctuations were too irregular and chaotic to indicate fully developed slugging. The presence of the tubes may have played a part in preventing slug flow. Further details of the experimental set-up and hydrodynamic measurements are given by George [1].

HEAT TRANSFER RESULTS

In-bed heat transfer coefficients, h_b, are shown in Figs. 1–3 for the three particle sizes used in this work. In agreement with previous studies [4–6], h_b decreased with increasing $\bar{d}_p$ in the 100–900 μm range covered. The shapes of the curves, with h_b increasing to a maximum value, h_{max}, and then declining with increasing superficial gas velocity, is also in agreement with many previous studies. Replicate experiments, shown by pairs of closely spaced data points, show good reproducibility.

Table 1. Properties of silica sand (density 2630 kg/m^3)

Designation	Size range (μm)	$\bar{d}_p$ (μm)	U_{mf} (m/s)	ε_{mf}
Small	30–272	102	0.015	0.452
Intermediate	200–920	470	0.17	0.410
Large	360–1430	890	0.62	0.498

Experimental heat transfer coefficients for the smallest particles and the instrumented tube in each of the three lowest rows appear in Fig. 1 together with predictions from some of the more popular correlations appearing in the literature. Heat transfer is seen to be more favourable in the second row than in the first, but the difference is less than 10%. The third row, on the other hand, gives somewhat lower values of h_b. These small differences are probably due to the complex trajectories and breakup of bubbles within the tube bundle. The h_{max} correlation of [7] is seen to give good predictions in this case. The Wender and Cooper [8] correlation, derived for vertical tubes but commonly used for horizontal tubes with the coefficient C_R taken as unity, overestimates h_b and does not predict a maximum. The recent correlations due to [10] and [11] (not shown) both overestimate h_b by large factors, whether ε is obtained from correlations [11], [12] or experimentally as

$$\varepsilon = [H - H_{mf}(1 - \varepsilon_{mf})]/H. \qquad (1)$$

Evidently, all these correlations must be applied with caution for particles outside the ranges for which they were derived.

Data for the 470 μm sand appear in Fig. 2. In this case, the lowest row gives the highest h_b values. A single tube, at the height normally occupied by the first row, gave coefficients about 10% lower. The smaller horizontal pitch (2.75 D_T) gave somewhat bigger values of h_b than the larger pitch (3.75 D_T), but the difference was relatively small. This trend is consistent with recent measurements by [13]. However, previous work [14, 15] also indicates that h_b decreases if tubes are brought so close together that particles cannot move freely through the gaps separating the tubes. The effects of tube spacing and position appear to decrease with increasing gas flow rate. The correlations shown in Fig. 2 do not distinguish between tubes in different rows or allow for any influence of tube spacing. The Gelperin *et al.* [7] correlation again gives a reasonable prediction of h_{max}. Surprisingly, both the Wender and Cooper [8] and Andeen and Glicksman [10] correlations give better predictions when ε is based on the equation of Leva [12] rather than experiment. The predictions of [8] are further improved if C_R is taken as 1.2. None of the other predictions are very successful.

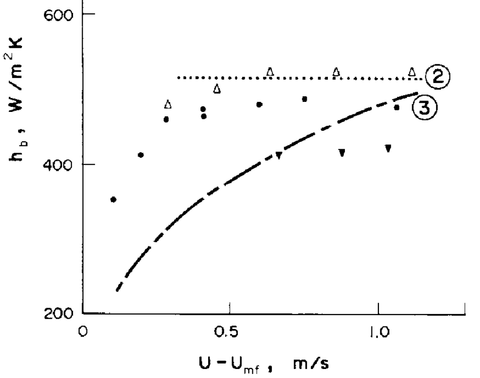

FIG. 1. Experimental results for 102 μm particles. Horizontal pitch = 3.75 D_T. Correlation lines: (1) [17], C_R = 1.0; (2) [7], h_{max}; (3) [9].

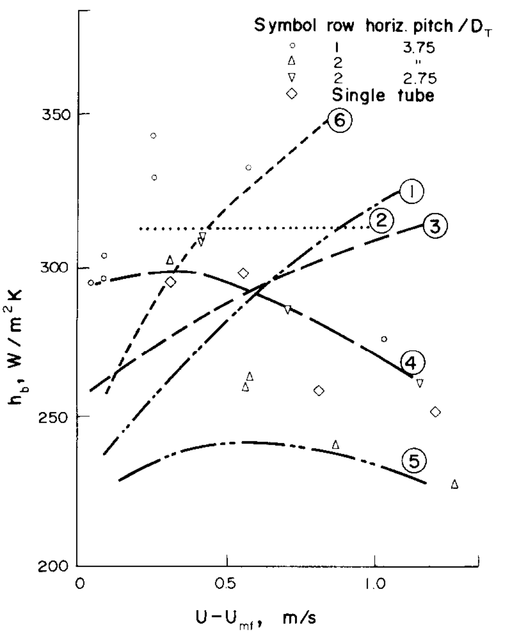

FIG. 2. Experimental results for 470 μm sand with correlations: (1) [10], measured ε; (2) [7], h_{max}; (3) [8], C_R = 1.0, measured ε; (4) [8], C_R = 1.2, ε from [12]; (5) [10], ε from [12]; (6) [11].

For $\bar{d}_p = 890\,\mu$m, the results plotted in Fig. 3 show that there is little effect of tube position on h_b. Agreement with the correlation of [10] is now within $\pm$ 5%. The other correlations shown all tend to overestimate h_b.

CONCLUSION

The experimental heat transfer measurements confirm a number of qualitative treads of previous studies including the shape of the h_b vs U curves, the effect of mean particle size, the insensitivity of the results to position within the tube bundle, and the relatively minor effect of tube spacing for the range investigated. However, commonly used heat transfer correlations show wide variation with each other and generally poor agreement with the experimental data. Improved methods are needed for predicting heat transfer to tubes in fluidized beds of large scale operated at high gas velocities.

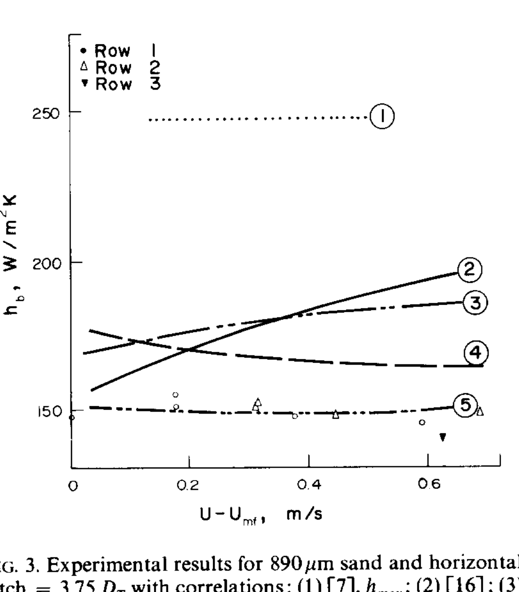

FIG. 3. Experimental results for 890 μm sand and horizontal pitch = 3.75 D_T with correlations: (1) [7], h_{max}; (2) [16]; (3) [10], ε from [12]; (4) [8], C_R = 1.0; (5) [10], measured ε.

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